Micro 2021 Paper

1)

(a)

Utilitarian criterion = max the sum of utilities.

$$\max \; x_A y_A^3 + 16(8-x_A)(8-y_A)$$

focs:

$$3x_A y_A^2 + 16(8-x_A) = 0 \qquad : \frac{\partial}{\partial y_A}$$
$$y_A^3 + 16(8-y_A) = 0 \qquad : \frac{\partial}{\partial x_A}$$

$(x_A, y_A) = (2,4)$ satisfies these focs

socs :

$$\cancel{6x_A y_A \qquad 3y_A^2 - 16}$$

$$\cancel{3y_A^2 - 16 \qquad 2y_A^2}$$

$$H = \begin{pmatrix} 0 & 3y_A^2 - 16 \\ 3y_A^2 - 16 & 6x_A y_A \end{pmatrix}$$

max iff H is ~~po~~ negative (semi)- definite.

$$\text{Det}(H) \;@\; (2,4) = 0 \cdot 6(2)(4) - (3\cdot 4^2 - 16)(3 \cdot 4^2 - 16)$$

~~⇒ indefinite ∴ not a max.~~

leading minors are 0 , -1024

$\Rightarrow$ indefinite $\therefore$ not a max.

Notice $(x_A, y_A) = (8,8)$ has higher utility than $(2,4)$ since

$$u = 8 \cdot 8^3 + 0 \cdot 0 \cdot 16 = 8^4 = 4096 > 2 \cdot 4^3 + 16 \cdot 6 \cdot 4 = 512$$

Pareto optimal?

Yes. : ① $MRS_A = MRS_B$

② endowments are fully exhausted

∴ lies on contract curve (set of pareto optimal allocations)

(b)

$(x_A, y_A) = (2,4)$, $(x_B, y_B) = (6,4)$ ~~$(w_A^x, w_B^x) = (8,0)$, (w_A^y, w_B^y)~~ =

$w_A = (8, w_A^y)$

$w_B = (0, w_B^y)$

$$MRS_A = -\frac{\frac{\partial u}{\partial x_A}}{\frac{\partial u}{\partial y_A}} = -\frac{y_A^{\cancel{3}}}{3x_A\cancel{y_A^2}} = -\frac{y_A}{3x_A} \quad @ (2,4) = -\frac{2}{3}$$

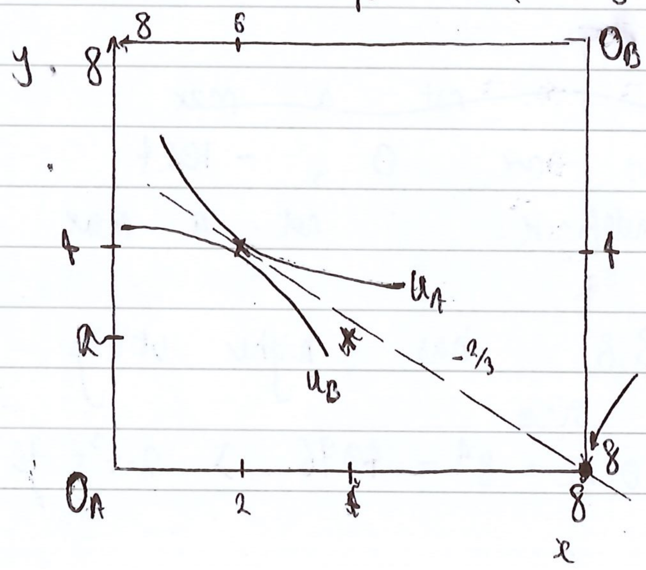
$p_x x_A + y_A = p_x \cdot 8 + w_A^y \qquad p_x = m\, 2/3$

$$y_A = -\frac{2}{3} \cdot x_A + \frac{2}{3} \cdot 8 + w_A^y$$

@ (2,4)

$$4 = -\frac{2}{3} \cdot 2 + \frac{2}{3} 8 + w_A^y \qquad \boxed{\begin{array}{l} w_A^y = 0 \\ w_B^y = 8 \end{array}}$$

∴ transfer all 8 from A to B.



(C)

Rawlsian : $\max\left[\min\{u_A, u_B\}\right]$

(i)

$u_A = u_B$

① Suppose $u_A > u_B$ then :
- transfer some goods to B in order to
$$\max\left[\min\{u_B, u_A\}\right]$$

② Suppose $u_A < u_B$ then :
- transfer some goods to A in order to $\max\left[\min\{u_A, u_B\}\right]$

Optimal point = no further redistribution
$\Rightarrow u_A = u_B$

(ii)

If $MRS_A \neq MRS_B$ then not pareto efficient, hence we could make one agent better off without making any worse off.

Hence we could make the agent with lower u better off ∴ improve on allocation ∴ cannot be optimal for Rawlsian.

(iii)

Must exhaust endowment.

2)

(a)

① College A:

$$\max \quad 42y^{1/2} - y^2 + M_A^* - 20h \qquad \text{s.t.} \quad y = 2h$$

$$\max_y \quad 42y^{\cancel{1/2}} - y^2 + M_A^* - 10y$$

foc:

$$42 - 2y - 10 = 0$$

$$\boxed{y = 16} \qquad \boxed{h = 8}$$

soc:

$$-2 < 0 \quad \therefore \quad \text{maximum.}$$

② Pareto efficient:

• Since Q-L utility max sum = pareto optimal.

$$\max_y \quad 42y - y^2 + M_A^* - 10y + \frac{1}{4}y^2 + 4y + M_B^*$$

foc:

$$42 - 2y - 10 + \frac{1}{2}y + 4 = 0$$

$$\boxed{y = 24} \qquad \boxed{h = 12}$$

soc:

$$-2 + \frac{1}{2} = -1.5 < 0 \quad \therefore \quad \text{maximum}$$

①:
$$u_A = 42 \cdot 16 - 16^2 + M_A^* - 10 \cdot 16 = 256$$
$$u_B = \frac{1}{4} 16^2 + 4 \cdot 16 + M_B^* = 128$$

②:
$$u_A = 42 \cdot 24 - 24^2 + M_A^* - 10 \cdot 24 = 198$$
$$u_B = \frac{1}{4} 24^2 + 4 \cdot 24 + M_B^* = 240$$

Can't pareto rank.

(b)

A needs to be transferred at least $256 - 192 = \cancel{80}\ 64$
B can transfer at most $\cancel{196}\ 240 - 128 = \cancel{68}\ 112$

$\therefore$ B transfers to A x : $64 \leq x \leq 112$

(how much depends on negotiation, if makes take-it-or-leave-it offer then she can offer > 64 units, or if A does then as for < 112).

(c)

- Quasi-linear utility
  = sum $\Rightarrow$ pareto optimization
  (Samuelson theorem)
  = partial eq. analysis.
- Pos. externality = inefficiency.

(3)

(a)

$$p(q) = \alpha - q_1 - q_2$$

firm 1:

$$\max_{q_1} \pi_1(q_1, q_2) = (\alpha - q_1 - q_2) q_1 - c_1 q_1$$

foc:

$$0 = \alpha - 2q_1 - q_2 - c_1$$

$$\boxed{BR_1(q_2) = q_1^* = \frac{\alpha - q_2 - c_1}{2}}$$

firm 2:

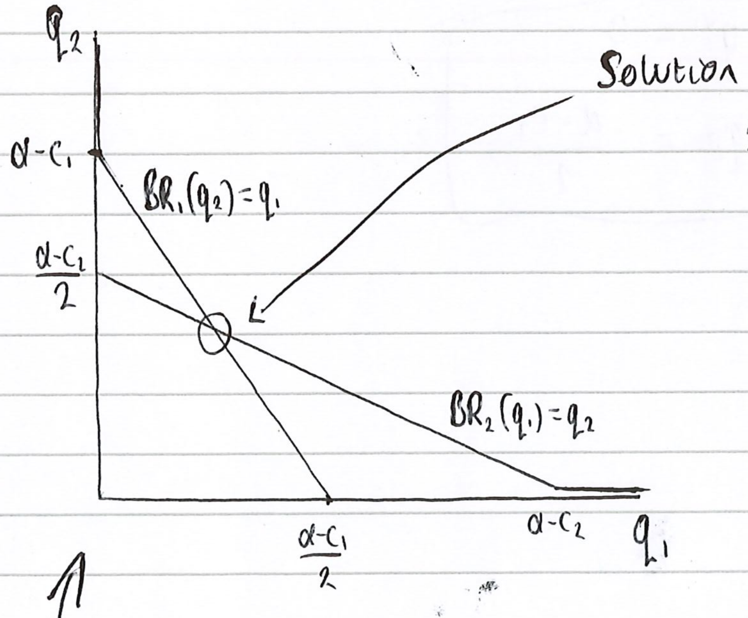
$$\max_{q_2} \pi_2(q_1, q_2) = (\alpha - q_1 - q_2) q_2 - c_2 q_2$$

foc:

$$0 = \alpha - q_1 - 2q_2 - c_2$$

$$\boxed{BR_2(q_1) = q_2^* = \frac{\alpha - q_1 - c_2}{2}}$$

(b)



Solution:

$$2q_1^* = \alpha - c_1 - \frac{\alpha - q_1 - c_2}{2}$$

$$4q_1^* = 2\alpha - 2c_1 - \alpha + q_1 + c_2$$

$$3q_1^* = \alpha - 2c_1 + c_2$$

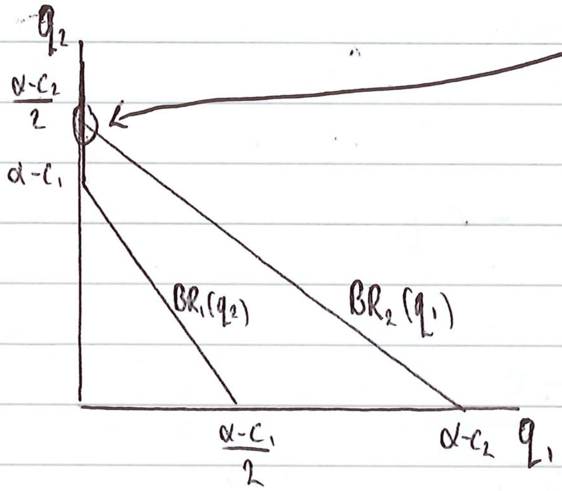
$$\boxed{q_1^* = \frac{\alpha + c_2 - 2c_1}{3}}$$

$$\boxed{q_2^* = \frac{\alpha + c_1 - 2c_2}{3}}$$

here $\alpha - c_1 > \frac{\alpha - c_2}{2}$

②

$$\alpha - c_1 < \frac{\alpha - c_2}{2}$$



Solution : $\boxed{q_1^* = 0 \quad q_2^* = \frac{\alpha - c_2}{2}}$

∴ firm 2 (more efficient) = monopolist.

(b)

Cournot - Nash ① :

$$\boxed{\begin{aligned} q_1^* &= \frac{\alpha + c_2 - 2c_1}{3} \\ q_2^* &= \frac{\alpha + c_1 - 2c_2}{3} \end{aligned}}$$

Cournot - Nash ② :

$$\boxed{\begin{aligned} q_1^* &= 0 \\ q_2^* &= \frac{\alpha - c_2}{2} \end{aligned}}$$

(4)

(a)

reservation utility $= \sqrt{36} - 2 = 4$

Observable ($\therefore$ IR only)

$\sqrt{w_0} - 0 = 4$

$\boxed{w_0 = 16}$

$\sqrt{w_1} - 1 = 4$

$\boxed{w_1 = 25}$

Which is optimal?

$E[\pi - w_1 | e=1] = \frac{1}{2} \cdot 70 + \frac{1}{2} 30 - 25 = 25$

$E[\pi - w_0 | e=0] = \frac{1}{4} 70 + \frac{3}{4} 30 - 16 = 24$

$E[\pi - w | e=1] > E[\pi - w | e=0]$

$\therefore$ pay $w_1 = 25$ (optimal)

~~(b) Unobservable:~~

~~• Induce low effort by $w_0 = 16$~~

~~• Induce high effort:~~

~~$V_i = \sqrt{w(\pi_i)}$~~

~~IR: $\frac{1}{2} V_H + \frac{1}{2} V_L \geq 4$~~

~~① $V_H + V_L \geq 8.$~~

~~IC: $\frac{1}{2} V_H + \frac{1}{2} V_L - 1 \geq \frac{1}{4} V_H + \frac{3}{4} V_L - 0$~~

~~② $2V_H + 2V_L - 4 \geq V_H + 3V_L - 0$~~

~~$V_H - V_L \geq 4$~~

~~① + ②~~

~~$2V_H \geq 12$ $V_H \geq 6$ $V_L \leq$~~

(b) Observable effort

- Induce $e=0$ by $w_0 = 16$ since IR constraint holds
- Induce $e=1$

$$V_i = \sqrt{w(\pi_i)} \quad i \in 1,2$$

~~IR:~~

IR:

$$\tfrac{1}{2}V_H + \tfrac{1}{2}V_L \geq 4$$

$$① \quad V_H + V_L \geq 10$$

$$\mathbb{E}[v(w(\pi))|e=1] - g(e=1) \geq \bar{u}$$

IC:

$$\tfrac{1}{2}V_H + \tfrac{1}{2}V_L - 1 \geq \tfrac{1}{4}V_H + \tfrac{3}{4}V_L - 0$$

$$\tfrac{1}{4}V_H - \tfrac{1}{4}V_L \geq 1$$

$$② \quad V_H - V_L \geq 4$$

① + ②

$$2V_H \geq 14 \qquad V_H \geq 7$$

$$V_L \leq 3$$

$$w(\pi_H) = V_H^2 = 49$$

$$w(\pi_L) = V_L^2 = 9$$

$$\mathbb{E}[w(\pi)|e=1] = \tfrac{1}{2}49 + \tfrac{1}{2}9 = 29$$

ASK

What is optimal?

Agency cost?

$$\mathbb{E}[\pi - w|e=1] = \tfrac{1}{2}\cdot 70 + \tfrac{1}{2}\cdot 30 - \tfrac{1}{2}\cdot 49 - \tfrac{1}{2}\cdot 9$$

$$= 21$$

$= 4$

$(\mathbb{E}[w(\pi)|e=1] - w_H$

$29 - 25$

$= 4)$

$$\mathbb{E}[\pi - w|e=0] = \tfrac{1}{4}\cdot 70 + \tfrac{3}{4}\cdot 30 - 16$$

$$= 24$$

Cost of effort unobservable
- RP of risk averse agents.

$$\mathbb{E}[\pi - w|e=0] > \mathbb{E}[\pi - w|\pi=1]$$

∴ induce $e=0$ and pay $w(e=0) = 16$

10)

(a)

Should add these.

$$L_W = \left[\tfrac{2}{5}, \tfrac{3}{10}, \tfrac{3}{10}; 36, 36, 25\right] = \left[\tfrac{7}{10}, \tfrac{3}{10}; 36, 25\right]$$

$$L_R = \left[\tfrac{2}{5}, \tfrac{3}{10}, \tfrac{3}{10}; 64, 36, 9\right]$$

$$L_S = \left[\tfrac{2}{5}, \tfrac{3}{10}, \tfrac{3}{10}; 100, 36, 0\right]$$

$$\mathbb{E}V[L_W] = \tfrac{2}{5}\cdot 36 + \tfrac{3}{10}\cdot 36 + \tfrac{3}{10}\cdot 25 = 32.7 \checkmark$$

$$\mathbb{E}V[L_R] = \tfrac{2}{5}\cdot 64 + \tfrac{3}{10}\cdot 36 + \tfrac{3}{10}\cdot 9 = 39.1 \checkmark$$

$$\mathbb{E}V[L_S] = \tfrac{2}{5}\cdot 100 + \tfrac{3}{10}\cdot 36 + 0 = 50.8 \checkmark$$

$$\mathbb{E}V[L_S] > \mathbb{E}V[L_R] > \mathbb{E}V[L_W]$$ ✓

soybeans has highest $\mathbb{E}V$.

$$u(x) = \sqrt{x}$$

$$\mathbb{E}U[L_W] = \tfrac{2}{5}\sqrt{36} + \tfrac{3}{10}\sqrt{36} + \tfrac{3}{10}\sqrt{25} = 5.7 \checkmark$$

$$\mathbb{E}U[L_R] = \tfrac{2}{5}\sqrt{64} + \tfrac{3}{10}\sqrt{36} + \tfrac{3}{10}\sqrt{9}) = 5.9 \checkmark$$

$$\mathbb{E}U[L_S] = \tfrac{2}{5}\sqrt{100} + \tfrac{3}{10}\sqrt{36} + 0 = 5.8 \checkmark$$

$$\mathbb{E}U[L_R] > \mathbb{E}U[L_S] > \mathbb{E}U[L_W]$$ ✓

if expected utility maximiser then should plant Rice.

- Divergence between $\mathbb{E}V$ and $\mathbb{E}U$ since Goku is risk averse ✓ $\left(\frac{\partial^2}{\partial x^2}\sqrt{x} = -\frac{1}{x^2} < 0 \ \forall x \ \therefore \text{Concave} \Rightarrow \text{risk averse}\right)$

hence although Soya has a high expected return it is very risky (returns 0 $\frac{3}{10}$ of the times) and Goku is averse to risk.

(soyabeans has larger variability)

(b)

Independence : $L_1 \succsim_r L_2$ iff $[p, 1-p; x, L_1] \succsim [p, 1-p; x, L_2]$

$EU[L_W] = 5.543$
$EU[L_R] = 5.743$
$EU[L_S] = 5.6431$

$\therefore$ yes would still grow rice.

$L_i = [\frac{2}{5}, \frac{3}{10}, \frac{3}{10}; x_i, 30, y_i]$

$= [\frac{3}{10}, \frac{7}{10}; 30, L_j]$

where $L_j = [\frac{4}{7}, \frac{3}{7}; x_i, y_i]$

for $i = \{W, R, S\}$ and $x_W = 36$ $x_R = 64$ $x_S = 100$
$y_W = 25$ $y_R = 9$ $y_S = 0$

hence all these can be written as compounds:

$L_W = [\frac{3}{10}, \frac{7}{10}; 36, [\frac{4}{7}, \frac{3}{7}; 36, 25]]$

$L_R = [\frac{3}{10}, \frac{7}{10}; 36, [\frac{4}{7}, \frac{3}{7}; 64, 9]]$

$L_S = [\frac{3}{10}, \frac{7}{10}; 36, [\frac{4}{7}, \frac{3}{7}; 100, 0]]$

if $L_R \succ_r L_S \succsim L_W$ then by independence changing 36 to 30 should not change this preference ordering.

Additivity of EU: implies that replacing $0.3 \cdot \sqrt{36}$ with $0.3 \cdot \sqrt{30}$ doesn't Δ ordering of EU since subtracts same constant from each EU.

$0.3 \cdot \sqrt{36} - 0.3\sqrt{30} = \frac{18 - 3\sqrt{30}}{10} \approx 0.157$

Group together things with same outcome.

(c)

$$L_{S+I} = \left[\frac{2}{5}, \frac{3}{10}, \frac{3}{10}; 100-\cancel{36}^{21}, 36-\cancel{36}^{21}, 0+\cancel{21}^{36-21}\right]$$

$$EU[L_{S+I}] = \frac{2}{5}\cdot\sqrt{79} + \frac{3}{10}\cdot\sqrt{15} + \frac{3}{10}\sqrt{15}$$

$$= 5.879$$

$$EU[L_{S+I}] = 5.879 < 5.9 = EU[L_R]$$ ✓

$\therefore$ Still will grow Rice.

(d)

$$L_{S+W} = \left[\frac{2}{5}, \frac{3}{10}, \frac{3}{10}; \frac{100+36}{2}, \frac{36+36}{2}, \frac{0+25}{2}\right]$$

$$EU[\ ] = \underline{\underline{6.16}}$$ ✓

- This is an example of risk ~~pooling~~. Sharing
- When they do this ~~the high risk~~ the risk is pooled and hence lowered.

- This lottery FOSD's L_R & every utility max. would prefer it.

obvious since return is higher or equal in every weather condition.

(could plot CDF).

(e)

$\lambda : w \qquad 1-\lambda : s$

~~$\max_{\lambda} \; \lambda \left[\frac{2}{5}\sqrt{36} + \frac{3}{10}\sqrt{36} + \frac{3}{10}\sqrt{25} \right] + [1-\lambda] [$~~

$$\mathbb{L}_{s+w} = \left[\frac{2}{5}, \frac{3}{10}, \frac{3}{10}, \frac{(1-\lambda)100 + \lambda 36}{1}, \frac{(1-\lambda)36 + \lambda 36}{1}, \frac{(1-\lambda)\cdot 0 + \lambda 25}{1} \right]$$

$$\max_{\lambda} \mathbb{E}U = \frac{2}{5}\sqrt{(1-\lambda)100 + \lambda 36} + \frac{3}{10}\sqrt{(1-\lambda)36 + \lambda 36} + \frac{3}{10}\sqrt{\lambda 25}$$

$$= \frac{2}{5}(100 - \lambda 64)^{\frac{1}{2}} + \frac{3}{10}(36)^{\frac{1}{2}} + \frac{3}{10}(\lambda 25)^{\frac{1}{2}}$$

foc:

$$0 = \frac{2}{5}\cdot(-64)\cdot\frac{1}{2}(100-64\lambda)^{-\frac{1}{2}} + \cancel{0} + \frac{3}{10}\cdot 25\cdot\frac{1}{2}(25\lambda)^{-\frac{1}{2}}$$

$$\frac{64}{5}\frac{1}{(100-64\lambda)^{\frac{1}{2}}} = \frac{15}{4}\frac{1}{(25\lambda)^{\frac{1}{2}}}$$

$$256\cdot(25\lambda)^{\frac{1}{2}} = 75(100-64\lambda)^{\frac{1}{2}}$$

$$256^2\cdot 25\lambda = 75^2(100-64\lambda)$$

$$256^2\cdot 25\lambda + 75^2\cdot 64\lambda = 75^2\cdot 100$$

$$\lambda = \frac{75^2\cdot 100}{256^2\cdot 25 + 75^2\cdot 64} = 0.2815$$

soc:

(definitely a max.)

- Objective function is concave hence we are considering a max.

(f)

- they all face the same pay off / risk (identical lotteries) $\therefore$ no benefit to risk ~~pooling~~ sharing ✓

- They may face different probabilities $\therefore$ benefits to risk sharing.

outcomes independent

$$L_W = [0.7, 0.3; 36, 25]$$

together:

$$L_{2xW} = \left[0.7 \times 0.7, 2 \times 0.7 \times 0.3, 0.3 \times 0.3; 36, \frac{36+25}{2}, 25\right]$$

(both success) ↑ both Good/ok

(one success one fail) ↑ one good/ok one bad

(both fail)

$= \boxed{\text{Risk Pooling}}$

$$L_{2xW} = [0.49, 0.42, 0.09; 36, 30.1, 25]$$

L_{2xW} SOSDs L_W, should use a CDF to show crosses once from below.

Microeconomics 2020

(1)

(a)

Pareto efficient if:

① exhausts endowment ✓

② $MRS_A = MRS_B$

$$MRS_A = -\frac{\frac{\partial u_A}{\partial x_A}}{\frac{\partial u_A}{\partial y_A}} = -\frac{2\frac{1}{x_A}}{\frac{1}{y_A}} = -\frac{2y_A}{x_A}$$

$$MRS_B = -\frac{x_B^{-1}}{2y_B^{-1}} = -\frac{y_B}{2x_B}$$

$MRS_A = MRS_B$: $\frac{2y_A}{x_A} = \frac{y_B}{2x_B}$

$A = (4,4)$ $B = (11,8)$

$$\frac{2\cdot 4}{4} = 2 \qquad \cancel{\frac{11}{2\cdot 8} = \frac{11}{16}} \qquad \frac{8}{2\times 11} = \frac{8}{22}$$

$\therefore$

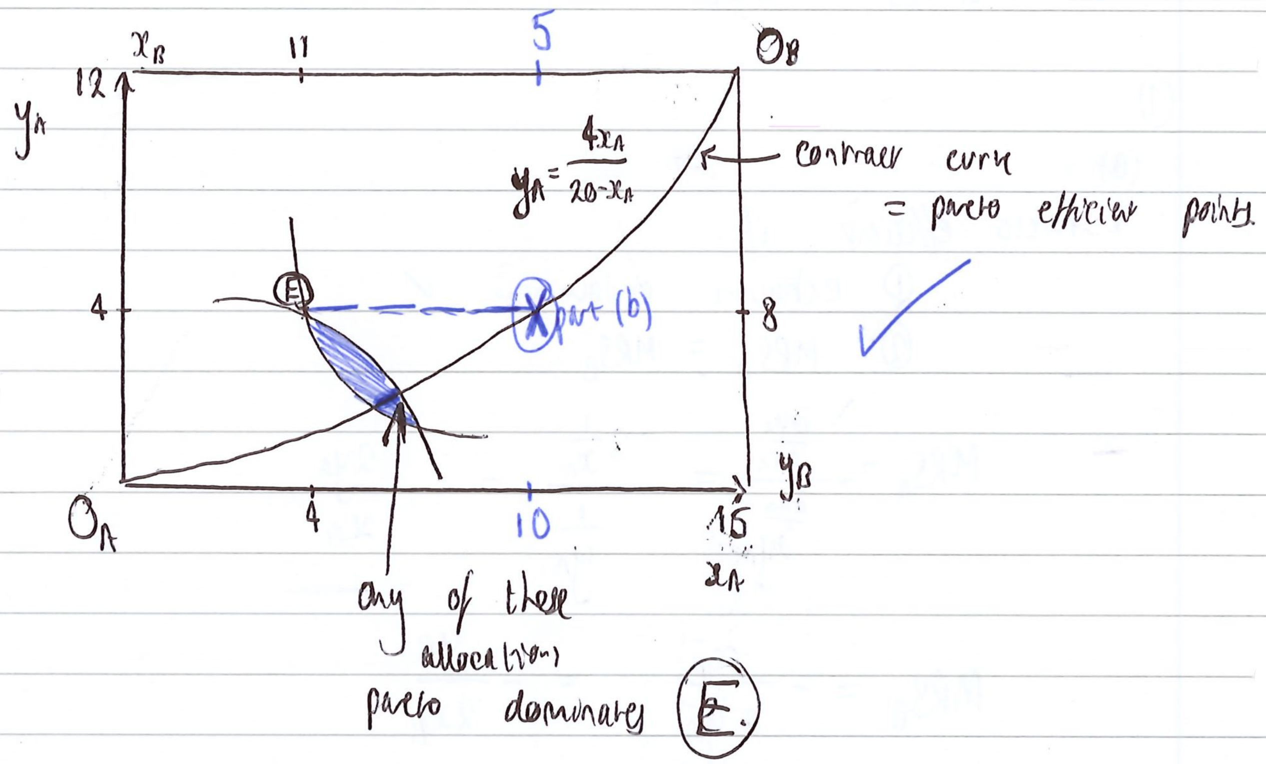
$MRS_A \neq MRS_B$

Contract curve = $y_A = \frac{x_A}{2} \frac{12-y_A}{2(15-x_A)}$

$$2(15-x_A)\frac{2}{x_A} y_A = 12 - y_A.$$

$$y_A = \frac{12}{2(15-x_A)\frac{2}{x_A}+1} = \frac{12}{\frac{60}{x_A} - \frac{4x_A}{x_A} + 1} = \frac{12}{\frac{60}{x_A} - 3}$$

$$\boxed{y_A = \frac{12x_A}{60-3x_A}}$$



(b)

let $P_y = 1$

$\cancel{P_x x_A + P_y y_A = P_x \cdot w_A^x + 4}$

$\cancel{y_A = -P_x x_A + P_x \cdot w_A^x + 4}$

$\cancel{y_B = -P_x x_B + P_x w_B^x + 8}$

$\cancel{y_A + y_B =}$

contract curve = $y_A = \frac{4x_A}{20-x_A}$

$y_A = 4 \quad \therefore \quad 20 - x_A = x_A.$

$x_A = 10$

$\therefore \ x_B = 5$

Pareto efficient allocation : $\boxed{\begin{array}{l} A = (10, 4) \\ B = (5, 8) \end{array}}$

$\therefore$ endowments:

$$p_x x_A + y_A = p_x w_A^x + 4$$

$$-p_x = MRS_A = MRS_B$$

$$-p_x = -\frac{2 \cdot 4}{10} = -\frac{8}{10} = -\frac{4}{5} \quad ✓$$

$$= -\frac{8}{2.5} = -\frac{8}{10} = -\frac{4}{5} \quad ✓$$

$$\frac{4}{5} \cdot 10 + 4 = \frac{4}{5} w_A^x + 4$$

$$w_A^x = \frac{5}{4}\left(\frac{4}{5}10 + \cancel{4} - \cancel{4}\right)$$

$$\boxed{w_A^x = 10 \qquad w_B^x = 5}$$

$\therefore$ transfer 6 of x to A. ✓

(c)

(a)

$$u_A(4,4) = \ln(4^2 \times 4) = 3\ln(4) = 4.158$$

$$u_B(11,8) = \ln(11 \times 8^2) = 6.556$$

(b)

$$u_A(10,4) = \ln(10^2 \times 4) = 5.99$$

$$u_B(5,8) = \ln(5 \times 8^2) = 5.768.$$

$\therefore$ no cannot be pareto ranked ✓
since (a) is strictly better for A
and (b) strictly better for B.

(d)

$$\max \quad 2\ln(x^A) + \ln y^A + \ln x^B + 2\ln y^B$$

$$\text{s.t.} \quad x_A + x_B = 15 \qquad y_A + y_B = 12$$

$$\max \quad 2\ln(x_A) + \ln y_A + \ln(15 - x_A) + 2\ln(12 - y_A) \checkmark$$

foc:

$$\text{wrt } x_A = \frac{2}{x_A} - \frac{1}{15 - x_A} = 0 \quad ① \checkmark$$

$$\text{wrt } y_A = \frac{1}{y_A} + \frac{-2}{12 - y_A} = 0 \quad ② \checkmark$$

$$\frac{2}{x_A} = \frac{1}{15 - x_A} \implies 30 - 2x_A = x_A$$

$$\boxed{x_A = 10 \quad x_B = 5} \checkmark$$

$$12 - y_A = 2y_A \implies \boxed{y_A = 4 \quad y_B = 8} \checkmark$$

soc:

$$\cancel{H = \begin{pmatrix} \frac{1}{x^2} & 0 \\ 0 & \end{pmatrix}}$$

$$-(15 - x_A)^{-1}$$

$$+1 \cdot -1 \, (15 - x_A)$$

$$H = \begin{bmatrix} -\frac{2}{x^2} - \frac{1}{(15 - x_A)^2} & 0 \\ 0 & -\frac{1}{y_A^2} - \frac{2}{(12 - y_A^2)} \end{bmatrix}$$

$$|H| = \left[-\frac{2}{x^2} - \frac{1}{(15 - x_A)^2}\right] \cdot \left(-\frac{1}{y_A^2} - \frac{2}{(12 - y_A)^2}\right) < 0 \quad \therefore \max.$$

$\max \left[\min \{ u_A , u_B \} \right]$

$\uparrow$ implies $u_A = u_B$ since if $u_A > u_B$ $\therefore$ optimal to transfer some good to B and vice - versa if $u_A < u_B$

$u_A (10, 4) = 5.49$

$u_B (5, 8) = 5.708$

$\therefore$ yes this is not optimal for Rawlsian.

(2)

(a)

Nash eq. = strat. ✓ profile $S = (S_1, S_2 \ldots S_n)$ s.t. each players strategy is a best response to the strategies of the other players.

Nash eq:

① (Increase, Maintain) ✓

② (Maintain, Increase) ✓

(Mixed strategy):

O plays I with p_I^O

Keep L indifferent:

$$-1p_I^O + 0(1-p_I^O) = -4p_I^O + 2(1-p_I^O)$$

$$3p_I^O = 2 - 2p_I^O$$

$$5p_I^O = 2$$

$$\therefore \quad p_I^O = \frac{2}{5}$$

L plays I with p_I^L

Keep O indifferent:

$$0(1-p_I^L) + -1p_I^L = 2(1-p_I^L) - 4p_I^L$$

$$3p_I^L = 2 - 2p_I^L$$

$$p_I^L = \frac{2}{5}$$

$\therefore$

③ (O play I with $\frac{2}{5}$ prob. ✓, L plays I with $\frac{2}{5}$ prob. ✓)

$\left(\frac{3}{5}M + \frac{2}{5}I \;,\; \frac{3}{5}M + \frac{2}{5}I\right)$ $\boxed{\text{Expected payoff} = -\frac{2}{5} \text{ each}}$

(b)

SPE : Nash eq. which induces a Nash eq. in each subgame

(c)

$$PV_{maintain} = 0 + 0\delta + 0\delta^2 + \dots$$

$$PV_{cheat} = 2 + -\overset{1}{\cancel{4}}\delta - \overset{1}{\cancel{4}}\delta^2 + \dots$$

maintain if

$$0 \geq 2 - \frac{\overset{1}{\cancel{4}}\delta}{(1-\delta)}$$

$$\frac{\overset{1}{\cancel{4}}\delta}{1-\delta} \geq 2$$

$$\cancel{4}\delta \geq 2 - 2\delta$$

$$3\delta \geq 2$$

$$\cancel{6\delta \geq 2}$$

$$\boxed{\cancel{\delta \geq 2/3}} \qquad \therefore \quad \delta \geq 2/3$$

Alternative punishment : play mixed strategy equilibrium.

NOTE :

Punishment must be a Nash Equilibrium!!

they'll always best respond

(Can't play a non-nash punishment)

(3)

(a) CE : amount of money which would be as good to the individual as playing the lottery

$(u(CE) = EU(\text{lottery}))$

if CE < wealth then don't play
if CE > wealth then do play
if CE = wealth the indifferent about playing

for a Lottery = $[p, (1-p) ; w-L, w+W]$

where w = wealth

(b)

$X = [\frac{1}{2}, \frac{1}{2} ; 6, 30]$ $EV[X] = 18$

$Y = [\frac{1}{2}, \frac{1}{2} ; 12, 20]$ $EV[Y] = 16$

$Z = [\frac{1}{2}, \frac{1}{2} ; 14, 22]$ $EV[Z] = 18$

~~Z = mean preserving spread of X~~

~~($EV(Z) = \frac{1}{2}12 + \frac{1}{2}22 = 27$ $EV[X] = 3 + 15 = 18]$)~~

~~Same~~ Sam is correct.

- Z is a mean preserving spread of X.
  ∴ it is true that for risk neutral
  $Z \sim_i X$
  it is also true that if she were risk-loving
  $X \succ_i Z$
- ∵ $EU[Z] > EU[Y]$ ∴ yes she must be risk averse.

(c)

$\mathbb{E}u(Y) = \frac{1}{2}\ln(12-a) + \frac{1}{2}\ln(20-a)$

$\mathbb{E}u(X) = \frac{1}{2}\ln(6) + \frac{1}{2}\ln(30) = \frac{1}{2}\ln 180 \approx 2.596$

?

$\mathbb{E}u(Y) = \frac{1}{2}\ln((12-a)(20-a)) = \cancel{\frac{1}{2}\ln(240-20a)}$

~~$Y \succ_s X$ iff $\frac{1}{2}\ln(240-20a) > \frac{1}{2}\ln(180)$~~

~~$240-20a > 180$~~

~~$a < 3$~~

~~if cost increa~~

$Y \succ_s X$ if $\frac{1}{2}\ln((12-a)(20-a)) \geq \frac{1}{2}\ln\cancel{240-20a}$ (180)

$(12-a)(20-a) > 180$

$a^2 - 32a + 60 > 0$ ✓

$a^2 - 32a + 60 = 0$ when $a = 30$ or 2

$\boxed{a < 2 \mid \cancel{a > 30}}$

hence $a > 2$ is problematic, so $a = 2$ = largest cost ↑.

doesn't make sense since $a > 30$ implies ~~tot~~ she always loses.

(3b)

For Janet : $\frac{1}{2}u(12) + \frac{1}{2}u(20) > \frac{1}{2}u(6) + \frac{1}{2}u(30)$

FOR EU maximiser : $\frac{1}{2}u(14) + \frac{1}{2}u(22) > \frac{1}{2}u(12) + \frac{1}{2}(20)$

(+2 to each outcome.

if risk neutral : $X \sim Z$ since

$$\frac{1}{2}u(6) + \frac{1}{2}u(30) = \cancel{\frac{1}{2}u(180)}\ \frac{1}{2}u(36)$$

and

$$\frac{1}{2}u(14) + \frac{1}{2}u(22) = \frac{1}{2}u(36)$$

$$\frac{1}{2}u(6) + \frac{1}{2}u(30) = \frac{1}{2}u(14) + \frac{1}{2}u(22)$$

(same mean).

if risk loving : $X \succ_s Z$ since.

$$\frac{1}{2}u(6) + \frac{1}{2}u(30) > \frac{1}{2}u(14) + \frac{1}{2}u(22)$$

(mean preserving spread)

$\therefore$ She is risk averse IF her risk attitudes are independent to wealth.

(4)

(a)

Signal to receive wage = productivity
(type H want higher wage)

credible = L needs to be indifferent between getting education (signalling) + receiving high wage and not education + taking Low wage
then assume disutility of education.
∴ pick Low wage.

(b)

No signal

$w^* = \mathbb{E}(\Theta) = \frac{3}{5} \cdot 500 + \frac{2}{5} \cdot 400 = 460$

$w^* = 460$

(c)

Separating equilibrium.

~~$U_H = 500 - 60 = 440$: $w(\Theta_H) =$~~

~~$u_L = 400 =$~~

~~if L got education : $u_L = 500 - 80 = 420$~~

$= w(\Theta_H) = 500$, $w(\Theta_L) = 400$

$\therefore U_H = 500 - 60 = 440$

$U_L = 400$

if L got signal $U_L = 400 - 80 = 420$

$420 > 400$ ∴ not an ~~separating~~ equilibrium!

- signal not credible since it is in type-L's interest to imitate the signal + take $w(\Theta_H) = 500$.

$\therefore$ Eq = wage of 460 + education = 0 ✓

(10)

$$p(q) = 75 - q_1 - q_2$$

$$c(q_j) = 15 q_j \qquad j = 1, 2$$

(a)

$$\max_{q_1} \pi_1(q_1, q_2) = (75 - q_1 - q_2) q_1 - 15 q_1$$

foc:

$$75 - 2q_1 - q_2 - 15 = 0$$

$$BR_1(q_2) = q_1 = \frac{75 - q_2 - 15}{2} = \frac{60 - q_2}{2}$$

by symmetry:

$$q_2 = \frac{60 - q_1}{2}$$

Nash eq:

$$q_1^* = \frac{60 - \frac{60 - q_1^*}{2}}{2}$$

$$\cancel{2q_1^* = 120 - 60 + q_1^*}$$

$$\cancel{q_2^* = q_1^* = 60}$$

$$\cancel{\therefore \; q = 60 \qquad p(q) = 15}$$

$$2q_1^* = 60 - \frac{60 - q_1^*}{2}$$

$$4q_1^* = 120 - 60 + q_1^*$$

$$3q_1^* = 60$$

$$\boxed{q_1^* = q_2^* = 20}$$

$$\boxed{q = 40 \;\; p(q) = 35}$$

$$\boxed{\pi_1^* = \pi_2^* = 400}$$

(b)

Firm 1 invests.

$$\max \pi_1(q_1, q_2) = (75 - q_1 - q_2)q_1 - 9q_1$$

foc

$$75 - 2q_1 - q_2 - 9 = 0$$

$$\boxed{BR_1(q_2) = q_1 = \frac{66 - q_2}{2}}$$ ✓

$$\boxed{BR_2(q_1) = q_2 = \frac{60 - q_1}{2}.}$$

Nash eq:

$$2q_1^* = 66 - \frac{60 - q_1^*}{2}$$

$$4q_1^* = 132 - 60 - q_1^*$$

$$3q_1^* = 72 \qquad \boxed{q_1^* = 24} ✓ \qquad \boxed{\begin{array}{l} q = 42 \\ p(q) = 33 \end{array}} ✓$$

$$\boxed{q_2^* = 18} ✓$$

$$\boxed{\begin{array}{l} \pi_1 = 33 \cdot 24 - 9 \cdot 24 = 576 ✓ \\ \pi_2 = 33 \cdot 18 - 15 \cdot 18 = 324 ✓ \end{array}}$$

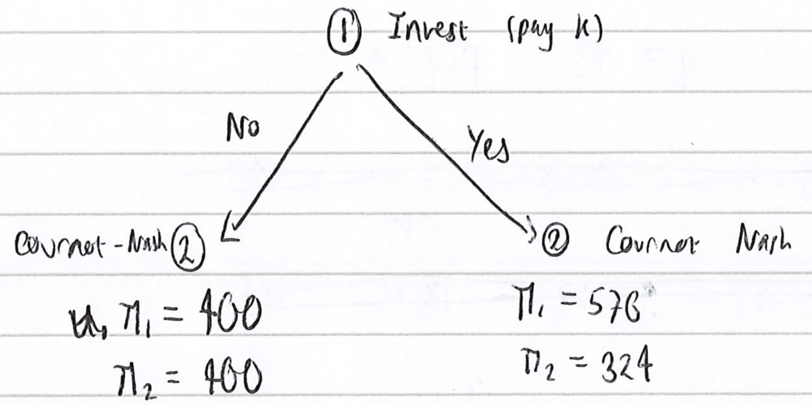
firm 1 now more efficient.

- prod costs ↓ for a given level of output
  - ∴ more aggresive
  - ⇒ push out BR curve, for any q_2
  - ∴ drives down q_2

(c)

① invest yes or no

② cournot - Nash.



$\therefore$ firm will invest for $K \leq 176$

$\therefore$ largest $K = 176$. ✓

K low enough =

Stage 1 : Firm 1 chooses to invest.

Stage 2 : Firm 1 & Firm 2 Best respond to play Cournot Nash with

$q_1^* = 24$

$q_2^* = 18$

Strategy profile $S = (S_1, S_2)$ S_1 = 1's strategy, S_2 = 2's strategy

$\boxed{S = (\text{Invest and } q_1 = 24,\ q_2 = 18)}$

X

$S = ($Invest and $q_1 = 24$, $q_2 = 20$ if no investment & $q_1 = 18$ if investment$)$

NEED ALL CASES !!

(d)

~~Not credible.~~

If ~~pt~~ $q_2 = 20$ $\quad BR_1(q_1) = \frac{66-20}{2} = 23 \quad q_1^* = 23$

$\therefore \; q = 43 \quad p(q) = 32$

$\cancel{\Pi_2(20, 23) = 32 \cdot 20 - 15 \cdot 20 = 340}$

$\cancel{\Pi_2(18, 24) = 33 \cdot 18 - 18 \cdot 15 = 324}$

$\Pi_1(20, 23) = 32 \cdot 23 - \cancel{15}^{9} \cdot 23 = \cancel{391} \; 529 - 150 < 400$

$\therefore$ not worth investing

But not credible since if firm 1 invests firm 2 would do better not following through.

$\therefore$ time inconsistent

(Because if firm 1 $q_1^* = 23$ then $BR_2 = \cancel{18}\,18.5$
$\therefore$ not Nash eq, Firm 2 isn't best responding))

Micro 2019

1

a

$$MRS = -\frac{\partial u / \partial x}{\partial u / \partial y} = -\frac{1/x_c}{2/y} = -\frac{y}{2x_c}$$

$$MRT = -\frac{\partial y}{\partial x} = -\frac{2}{3}(x_f)^{-1/3} = -\frac{2}{3(14-x_c)^{1/3}}$$

[in terms of y: $y^{3/2} = 14 - x_c$

$$MRT = -\frac{2}{3y^{1/2}}$$]

b

Efficient if:

a endowment exhausted:

@ $x = 6$, $y = 4$

$y^{3/2} = x_f$ $\quad x_f = 4^{3/2} = 8$

$8 + 6 = 14$ ✓

b

$MRS = MRT$

$$MRS = -\frac{4}{2 \cdot 6} = -\frac{1}{3}$$

$$MRT = -\frac{2}{3(14-6)^{1/3}} = -\frac{2}{3 \cdot 8^{1/3}} = -\frac{2}{3 \cdot 2} = -\frac{1}{3}$$ ✓

$\therefore$ efficient.

(c)

@ optimum $\quad MRS = MRT = -\frac{p_x}{p_y}$

$MRS = -\frac{1}{3} \qquad p_y = 1$

$\therefore \quad \boxed{p_x = \frac{1}{3}}$

$\pi = p_y \cdot y - p_x x_f = y - p_x(14 - x_c)$

$\pi = 4 - \frac{1}{3}(14 - 6)$

$\boxed{\pi = \frac{4}{3}}$

BC:

$p_x \cdot x + p_y \cdot y = \frac{1}{3} \cdot 6 + 4 = \boxed{6}$ (expenditure)

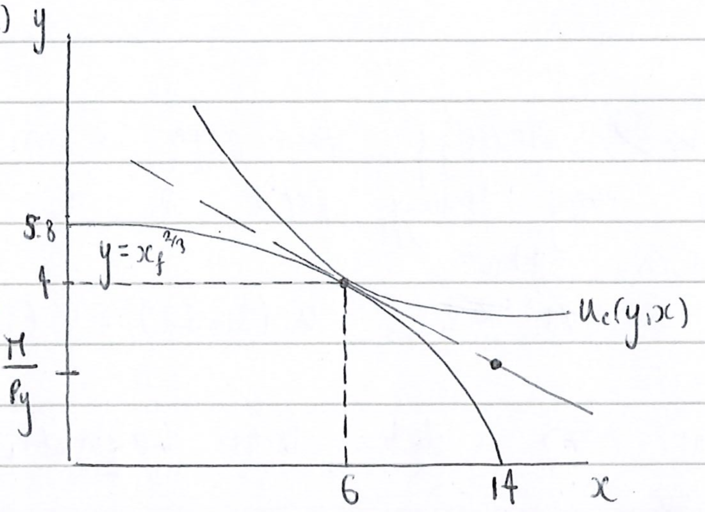
$14 \cdot p_x = 14 \cdot \frac{1}{3} = \boxed{\frac{14}{3}}$ (endowment)

$\therefore$

$6 = \frac{14}{3} + \frac{4}{3}$

$6 = 6$ ✓ $\therefore$ BC satisfied.

(d)



(2)

(a)

A (strictly) dominant strategy for player i is a (strict) BR to every strategy profile S_{-i} of the other players such that

$\forall S_{-i}$ and $\forall s_i' \neq s_i$, $u_i(s_i, s_{-i}) \geq u_i(s_i', s_{-i})$

If play dom. strat $\Rightarrow$ dom. strat. equilibrium $\Rightarrow$ Nash eq.

But Nash eq. $\not\Rightarrow$ dom strat. equilibrium

since not all games have dominant strategy eq. while all games have Nash.

(b) (1)

C

		Green	Red
R	Green	5, 5	6, 4
	Red	4, 6	5, 5

- ~~No pure strategy Nash eq. since~~ if ~~Green~~ C ~~plays green BR_R =~~
- ~~Two~~
- Pure strategy Nash equilibrium (Green, Green) since 5 > 4 and 6 > 5 (true for both players by symmetry)

(2)

C

		Blue	Yellow
R	Blue	3, 3	4, 2
	Yellow	2, 4	5, 5

- 2 Nash equilibria in pure strategy: (Blue, Blue) and (Yellow, Yellow)
- Mixed strategy eq:

R plays B with p_B to keep C indifferent

$3p_B + 4(1-p_B) = 2p_B + 5(1-p_B)$

$p_B - 4p_B + 5p_B = 5 - 4$ $\boxed{p_B = \frac{1}{2}}$

(c) Nash is useful when one Nash eq. (game 1) & rational players, but multiple Nash eq. + mixed strat. eq => hard to tell what rational ~~people~~ agents will do.

③

ⓐ

CE = amount of money which would be as good to an individual as playing the lottery

suppose lottery = either win or loss ∴ total wealth is either higher or lower after it.

$\mathbb{L} = [p, (1-p); w-l, w+m]$

∴ if $CE < W$ (initial wealth)

=> don't play

if $CE = W$

=> indifferent

if $CE > W$

=> play.

ⓑ

$\mathbb{L} = [\frac{1}{2}, \frac{1}{2}; 12, 0]$

$\mathbb{E}U = \frac{1}{2} 2(4+12)^{\frac{1}{2}} + \frac{1}{2} \cdot 2(0+4)^{\frac{1}{2}}$

$= (16)^{\frac{1}{2}} + 4^{\frac{1}{2}} = \boxed{6}$ $\boxed{\mathbb{E}U = 6}$

~~$u(CE) = \mathbb{E}U$~~

~~$CE = u^{-1}(\mathbb{E}U)$~~

~~$2(CE)^{\frac{1}{2}} = 6$~~ ~~$CE = \left(\frac{6}{2}\right)^2$~~ ~~$CE = 9$~~

• Indifferent between selling ticket at p and playing

$$u(4+p) = 6$$

$$2(4+p)^{1/2} = 6$$

$$4+p = (3)^2$$

$$\boxed{p = 5}$$

(c)

Would pay q s.t. $u(4) = u[L - p]$

$$u(4) = \frac{1}{2}2(4+12-q)^{1/2} + \frac{1}{2}2(4+0-q)^{1/2}$$

$$2(4)^{1/2} = u(4) = (16-q)^{1/2} + (4-q)^{1/2}$$

$$2^2 \cdot 4 = \left((16-q)^{1/2} + (4-q)^{1/2}\right)^2$$

$$\cancel{16} = (\cancel{16}-q) + (4-q) + 2(16-q)^{1/2}(4-q)^{1/2}$$

$$\overset{2}{\cancel{4}}-2q = \cancel{2}\left[(16-q)^{1/2}(4-q)^{1/2}\right]$$

$$(2-q)^2 = (16-q)(4-q)$$

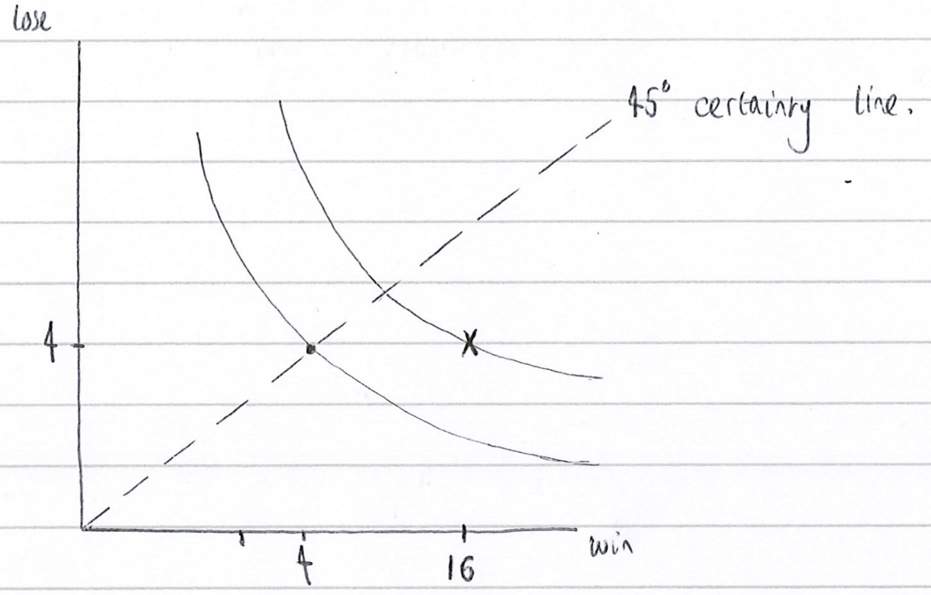
$$\cancel{q^2} - 2\cdot 2q + 4 = \cancel{q^2} - 16q - 4q + 64$$

$$20q - 4q = 60$$

$$16q = 60$$

$$\boxed{q = \frac{15}{4}}$$

(d)



- Janets lowest selling price (p) is higher than Sam's highest buying price (q).
  - To sell ~~sa~~ Janet would at minimum need to stay on her current ID curve (∴ receive CE) which is at (9,9)
  - To buy Sam would at minimum need to stay on her ID

    ?

- CARA => $p = q$
- But we have CRRA => DARA => distance between ID curves gets smaller as we move away from certainty.

④

ⓐ

Induce $e=1$ (contractible)

$\therefore$ participation constraint must bind

$$\sqrt{w_H} - 5(1) = 10$$

$$\boxed{w_H^* = 15^2 = 225}$$

- contractible effort $\Rightarrow$ can ignore IC constraint as agents are contracted to exert $e=1$
- IR must bind since u(high effort & high wage) $\geq$ reservation utility.

ⓑ Unobservable

$w_L = 36$, $w_H = ?$

IR:

$$\tfrac{3}{8} v(w_H) + \tfrac{5}{8} v(w_L) - 5 \geq 10$$

$v(\cdot) = \sqrt{\cdot}$

$w_L = 36$

$$\tfrac{3}{8}\sqrt{w_H} + \tfrac{5}{8}\sqrt{36} - 5 = 10$$

$$\tfrac{3}{8}\sqrt{w_H} = \frac{45}{4}$$

$$w_H = 900$$

IC:

$$\tfrac{3}{8}v(w_H) + \tfrac{5}{8}v(w_L) - 5 \geq \tfrac{1}{6}v(w_H) + \tfrac{5}{6}v(w_L) - 0$$

$$\tfrac{3}{8}\sqrt{900} + \tfrac{5}{8}\sqrt{36} - 5 = 10$$

$$\tfrac{1}{6}\sqrt{900} + \tfrac{5}{6}\sqrt{36} - 0 = 10$$

$\therefore$ IC holds!!

ⓒ

Agency cost = difference in ~~agency cost~~ expected wage when unobservable and high wage when effort is observable (for high effort) ii $\mathbb{E}[w] = \frac{5}{8} \cdot 36 + \frac{3}{8} \cdot 900 = 360$

Agency cost $= 360 - 225 = \boxed{135}$

Agency cost = Risk premium of ~~agent~~ lottery faced by agent exerting high effort.

(10)

A, B, C

$w_1 = (10, 0, 0)$

$w_2 = (0, 5, 0)$ $\quad\quad u(x_A, x_B, x_C) = x_A^{1/2} x_B^{1/4} x_C^{1/4}$

$w_3 = (0, 0, 20)$

(a) Each:

$$\max \quad x_A^{1/2} x_B^{1/4} x_C^{1/4} \quad \text{s.t.} \quad p_A x_A + p_B x_B + p_C x_C = \begin{cases} p_A 10 \\ p_B 5 \\ p_C 20 \end{cases}$$

(let $p_C = 1$)

- All will trade since convex ID curves $\Rightarrow$ asymptotic at 0 $\Rightarrow$ combinations always better than consuming just one good.

(1)

$$L = x_A^{1/2} x_B^{1/4} x_C^{1/4} - \lambda \left(p_A x_A + p_B x_B + \cancel{p_C} x_C - p_A \cdot 10 \right)$$

$$L_{x_A}: \quad \tfrac{1}{2} x_A^{-1/2} x_B^{1/4} x_C^{1/4} - \lambda p_A = 0$$

$$L_{x_B}: \quad \tfrac{1}{4} x_B^{-3/4} x_A^{1/2} x_C^{1/4} - \lambda p_B = 0$$

$$L_{x_C}: \quad \tfrac{1}{4} x_C^{-3/4} x_A^{1/2} x_B^{1/4} - \lambda = 0$$

$$p_A x_A + p_B x_B + x_C = p_A 10$$

$$\frac{\tfrac{1}{2} x_A^{-1/2} x_B^{1/4} x_C^{1/4}}{\lambda} x_A + \frac{\tfrac{1}{4} x_B^{-3/4} x_A^{1/2} x_C^{1/4}}{\lambda} x_B + \frac{\tfrac{1}{4} x_C^{-3/4} x_A^{1/2} x_B^{1/4}}{\lambda} x_C = 10 \cdot p_A$$

$$\tfrac{1}{2} x_A^{1/2} x_B^{1/4} x_C^{1/4} + \tfrac{1}{4} x_B^{1/4} x_A^{1/2} x_C^{1/4} + \tfrac{1}{4} x_A^{1/2} x_B^{1/4} x_C^{1/4} = 10 \cdot \lambda \cdot p_A$$

$$\boxed{x_A^{1/2} x_B^{1/4} x_C^{1/4} = 10 \cdot \lambda p_A}$$

$$x_A^{1/2} x_B^{1/4} x_C^{1/4} = 10 \cdot \lambda P_A$$

$$\lambda P_A = \frac{1}{2} x_A^{-1/2} x_B^{1/4} x_C^{1/4}$$

$$x_A^{1/2} \cancel{x_B^{1/4}} \cancel{x_C^{1/4}} = 10 \cdot \frac{1}{2} x_A^{-1/2} \cancel{x_B^{1/4}} \cancel{x_C^{1/4}}$$

$$\boxed{x_A^1 = 5}$$

$$\cancel{x_A^{1/2}} x_B^{1/4} \cancel{x_C^{1/4}} = 10 \cdot P_A \frac{1}{4} \frac{x_B^{-3/4} \cancel{x_A^{1/2}} \cancel{x_C^{1/4}}}{P_B}$$

$$\boxed{x_B^1 = \frac{5 P_A}{2 P_B}}$$

$$\cancel{x_A^{1/2}} \cancel{x_B^{1/4}} x_C^{1/4} = 10 \cdot P_A \cdot \frac{1}{4} x_C^{-3/4} \cancel{x_A^{1/2}} \cancel{x_B^{1/4}}$$

$$\boxed{x_C^1 = \frac{5}{2} P_A}$$

Trader 1:

$$\boxed{(x_A^1, x_B^1, x_C^1) = \left(5, \frac{5}{2}\frac{P_A}{P_B}, \frac{5}{2} P_A\right)}$$

By symmetry:

foc's : Trader 2 :

$$x_A^{1/2} x_B^{1/4} x_C^{1/4} = 5 \cdot \lambda \cdot P_B$$

$$x_A^{1/2} x_B^{1/4} x_C^{1/4} = 5 \cdot P_B \cdot \frac{1}{2} \frac{x_A^{-1/2} \cancel{x_B^{1/4}} \cdot \cancel{x_C^{1/4}}}{P_A} \qquad \boxed{x_A^2 = \frac{5}{2}\frac{P_B}{P_A}}$$

$$\cancel{x_A^{1/2}} x_B^{1/4} x_C^{1/4} = 5 \cdot P_B \cdot \frac{1}{4} \frac{x_B^{-3/4} \cancel{x_A^{1/2}} x_C^{1/4}}{P_B} \qquad \boxed{x_B^2 = \frac{5}{4}}$$

$$\cancel{x_A^{1/2}} \cancel{x_B^{1/4}} x_C^{1/4} = 5 \cdot P_B \cdot \frac{1}{4} x_C^{-3/4} \cancel{x_A^{1/2}} \cancel{x_B^{1/4}} \qquad \boxed{x_C^2 = \frac{5}{4} P_B}$$

foc's : Trader 3

$$x_A^{\frac{1}{2}} \cdot x_B^{\frac{1}{4}} x_C^{\frac{1}{4}} = 20 \cdot \lambda$$

$$x_A^{\frac{1}{2}} \cancel{x_B^{\frac{1}{4}}} \cancel{x_C^{\frac{1}{4}}} = 20 \cdot \frac{\frac{1}{2} x_A^{-\frac{1}{2}} \cancel{x_B^{\frac{1}{4}}} \cancel{x_C^{\frac{1}{4}}}}{p_A} \qquad \boxed{x_A^3 = \frac{10}{p_A}}$$

$$x_A^{\frac{1}{2}} x_B^{\frac{1}{4}} x_C^{\frac{1}{4}} = 20 \cdot \frac{\frac{1}{4} x_A^{\frac{1}{2}} x_B^{-\frac{3}{4}} x_C^{\frac{1}{4}}}{p_B} \qquad \boxed{x_B^3 = \frac{5}{p_B}}$$

$$\cancel{x_A^{\frac{1}{2}}} \cancel{x_B^{\frac{1}{4}}} x_C^{\frac{1}{4}} = 20 \cdot \frac{1}{4} \cancel{x_A^{\frac{1}{2}}} \cancel{x_B^{\frac{1}{4}}} x_C^{-\frac{3}{4}} \qquad \boxed{x_B^3 = 5}$$

hence:

$$\boxed{\begin{aligned} (x_A^1, x_B^1, x_C^1) &= \left(5, \frac{5}{2}\frac{p_A}{p_B}, \frac{5}{2}p_A\right) \\ (x_A^2, x_B^2, x_C^2) &= \left(\frac{5}{2}\frac{p_B}{p_A}, \frac{5}{4}, \frac{5}{4}p_B\right) \\ (x_A^3, x_B^3, x_C^3) &= \left(\frac{10}{p_A}, \frac{5}{p_B}, 5\right) \end{aligned}}$$

(b)

$$Z^A(p) = (5 - 10) + \left(\frac{5}{2}\frac{p_B}{p_A} - 0\right) + \left(\frac{10}{p_A} - 0\right)$$

$$= \boxed{\frac{5p_B + 20}{2p_A} - 5}$$

$$Z^B(p) = \left(\frac{5}{2}\frac{p_A}{p_B} - 0\right) + \left(\frac{5}{4} - 5\right) + \left(\frac{5}{p_B} - 0\right)$$

$$= \boxed{\frac{5p_A + 10}{2p_B} - \frac{15}{4}}$$

at equilibrium p^* $\quad Z^A(p^*) = Z^B(p^*) = 0$

$$0 = \frac{5P_B + 20}{2P_A} - 5$$

$$10P_A = 5P_B + 20$$

$$\boxed{\frac{P_A^*}{P_B^*} = 10}$$

$$0 = \frac{5P_A + 10}{2P_B} - \frac{15}{4}$$

$$\frac{30}{4}P_B = 5P_A + 10$$

$$\frac{30}{20}\frac{P_B}{P_A} = 10 \qquad \frac{P_B}{P_A} = \frac{200}{30} = \frac{20}{3}$$

(b) 20 traders, 10 type 1, 5 type 2, 5 type 3

$$Z^A(p_A, p_B) = 10(5-10) + 5\left(\frac{5}{2}\frac{p_B}{p_A} - 0\right) + 5\left(\frac{10}{p_A} - 0\right)$$

$$= -50 + \frac{25p_B}{2p_A} + \frac{50}{p_A}$$

$$Z^B(p_A, p_B) = 10\left(\frac{5}{2}\frac{p_A}{p_B} - 0\right) + 5\left(\frac{5}{4} - 5\right) + 5\left(\frac{5}{4p_B}p_B - 0\right)$$

$$= \frac{50p_A}{2p_B} + \frac{25}{4} - 25 + \frac{25}{4p_B}p_B$$

$$= 25\frac{p_A}{p_B} + \frac{25}{4p_B}p_B - \frac{75}{4}$$

@ Equilibrium $\mathbf{p}^* = (p_A^*, p_B^*)$ $\quad z^A(\mathbf{p}^*) = z^B(\mathbf{p}^*) = 0$

$$0 = \frac{25 p_B}{2 p_A} + \frac{50}{p_A} - 50 \qquad 0 = 25\frac{p_A}{p_B} + \frac{25}{4 p_B} \cancel{p_B} - \frac{75}{4}$$

$$0 = 25 p_B + 100 - 100 p_A \qquad 0 = 100 p_A + 100 - 75 p_B$$

$$+$$

$$0 = 25 p_B - 75 p_B + 200$$

$$50 p_B = 200$$

$$\boxed{p_B^* = 4}$$

$$\therefore \boxed{p_A^* = 2}$$

By Walras' law if K-1 markets clear then the K^{th} market clears.

$$\therefore \quad z^C(p_A^*, p_B^*) = 0$$

$$z^C(p_A, p_B) = \cancel{5} \; 10\left(\tfrac{5}{2} p_A\right) + 5\left(\tfrac{5}{4} p_B\right) + 5(5 - 20)$$

$$= 25 p_A + \frac{25}{4} p_B + -75$$

@ $(p_A, p_B) = (2, 4)$

$$= 25 \cdot 2 + \frac{25}{4} \cdot 4 - 75$$

$$= 50 + 25 - 75$$

$= 0$ as required.

(c)

New allocations: $(9, \frac{9}{4}, 9)$
$(1, \frac{1}{4}, 1)$
$(1, \frac{1}{4}, 1)$

Pareto efficient:

ⓐ exhausts allocation

ⓑ $MRS^k_{ij} = MRS^k_{ij} \quad \forall i,j \in \{A, B, C\} \quad \forall k \in \{1, 2, 3\}$

Good AB: $MRS^k_{A,B} = -\frac{\partial u/\partial A}{\partial u/\partial B} = -\frac{\frac{1}{2} x_A^{-\frac{1}{2}} x_B^{\frac{1}{4}} \cancel{x_C^{\frac{1}{4}}}}{\frac{1}{4} x_A^{\frac{1}{2}} x_B^{-\frac{3}{4}} \cancel{x_C^{\frac{1}{4}}}} = -\frac{2x_B}{x_A}$

Good BC: $MRS_{BC} = -\frac{\cancel{\frac{1}{4}} \cancel{x_A^{\frac{1}{2}}} x_B^{-\frac{3}{4}} x_C^{\frac{1}{4}}}{\cancel{\frac{1}{4}} \cancel{x_A^{\frac{1}{2}}} x_B^{\frac{1}{4}} x_C^{-\frac{3}{4}}} = -\frac{x_C}{x_B}$

Good AC: $MRS_{AC} = -\frac{\frac{1}{2} x_A^{-\frac{1}{2}} \cancel{x_B^{\frac{1}{4}}} x_C^{\frac{1}{4}}}{\frac{1}{4} x_A^{\frac{1}{2}} \cancel{x_B^{\frac{1}{4}}} x_C^{-\frac{3}{4}}} = -\frac{2x_C}{x_A}$.

ⓐ

AB: $MRS^1_{AB} = -\frac{2 \cdot \frac{9}{4}}{9} = -\frac{1}{2} \quad MRS^2_{AB} = -\frac{2\frac{1}{4}}{1} = -\frac{1}{2} \quad MRS^3_{AB} = -\frac{2\frac{1}{4}}{1} = -\frac{1}{2}$

$\therefore$ all $= -\frac{1}{2}$ ✓

BC: $MRS^1_{BC} = -\frac{9}{(9/4)} = -4 \quad MRS^2_{BC} = -\frac{1}{\frac{1}{4}} = -4 \quad MRS^3_{BC} = -\frac{1}{\frac{1}{4}} = -4$

$\therefore$ all $= -4$ ✓

AC: $MRS^1_{AC} = -\frac{2 \cdot 9}{9/4} = -2 \quad MRS^2_{AC} = -\frac{2 \cdot 1}{1} = -2 \quad MRS^3_{AC} = -\frac{2 \cdot 1}{1} = -2$

$\therefore$ all $= -2$ ✓

ⓑ

$9 \times 10 + 1 \times 5 + 1 \times 5 = 100$ ✓

$\frac{9}{4} \times 10 + \frac{1}{4} \times 5 + \frac{1}{4} \times 5 = 25$ ✓

$9 \times 10 + 1 \times 5 + 1 \times 5 = 100$ ✓

$\boxed{\therefore \text{Pareto efficient.}}$

Competitive eq. prices?

- ~~Must~~ price ratios = MRS.

$\cancel{\frac{2x_B}{x_A}} = \cancel{\frac{P_A}{P_B}}$ $\frac{P_A}{P_B} = \rightarrow \frac{1}{2}$ $\frac{P_B}{P_C} = 4$ $\frac{P_A}{P_C} = 2$

$P_C = 1$

$\boxed{\therefore \quad P_B = 4 \quad \text{and} \quad P_A = 2}$

$\boxed{(P_A, P_B, P_C) = (2, 4, 1)}$

Micro 2018

(1)

(a) $\max \quad pX - wL \quad s.t. \quad X = \frac{A}{\alpha} L^{\alpha}$

$$\max_{L} \quad p \frac{A}{\alpha} L^{\alpha} - wL$$

foc: $\quad pA L^{\alpha-1} - w = 0$

$$L^{\alpha-1} = \frac{w}{pA} \qquad L = \left[\frac{w}{pA}\right]^{\frac{1}{\alpha-1}}$$

$0 < \alpha < 1 \quad \therefore \quad \alpha - 1 < 0$ (not ideal)

$$\therefore \quad L = \left[\left(\frac{pA}{w}\right)^{-1}\right]^{\frac{-1}{-(\alpha-1)}} = \left[\left(\frac{pA}{w}\right)^{-1}\right]^{\frac{-1}{1-\alpha}}$$

$$\boxed{L = \left[\frac{pA}{w}\right]^{\frac{1}{1-\alpha}}}$$

$$\boxed{X = \frac{A}{\alpha}\left[\frac{pA}{w}\right]^{\frac{\alpha}{1-\alpha}}}$$

$$\pi = p\frac{A}{\alpha}\left[\frac{pA}{w}\right]^{\frac{\alpha}{1-\alpha}} - w\left[\frac{pA}{w}\right]^{\frac{1}{1-\alpha}}$$

$$\pi = \frac{pA}{\alpha}\left(\frac{pA}{w}\right)^{\frac{\alpha-1}{1-\alpha}}\left[\frac{pA}{w}\right]^{\frac{1}{1-\alpha}} - w\left[\frac{pA}{w}\right]^{\frac{1}{1-\alpha}}$$

$$(pA)^{\alpha-1} \qquad pA^{\frac{\alpha-1}{1-\alpha}} = pA^{-\frac{(1-\alpha)}{1-\alpha}} = \frac{1}{pA}$$

$$\pi = \frac{pA}{\alpha}$$

$$\Pi = p\frac{A}{\alpha}\left[\frac{pA}{w}\right]^{\frac{\alpha}{1-\alpha}} - w\left[\frac{pA}{w}\right]^{\frac{1}{1-\alpha}}$$

$$= \frac{pA}{\alpha}\left[\frac{pA}{w}\right]^{\frac{\alpha-1}{1-\alpha}}\left[\frac{pA}{w}\right]^{\frac{1}{1-\alpha}} - w\left[\frac{pA}{w}\right]^{\frac{1}{1-\alpha}}$$

$$\left[\frac{pA}{w}\right]^{\frac{\alpha-1}{1-\alpha}} = \left[\frac{pA}{w}\right]^{-\frac{(1-\alpha)}{1-\alpha}} = \frac{w}{pA}$$

$$\Pi = \frac{\cancel{pA}}{\alpha}\frac{w}{\cancel{pA}}\left[\frac{pA}{w}\right]^{\frac{1}{1-\alpha}} - w\left[\frac{pA}{w}\right]^{\frac{1}{1-\alpha}}$$

$$\Pi = \left[\frac{pA}{w}\right]^{\frac{1}{1-\alpha}}\left\{\frac{w}{\alpha} - w\right\}$$

$$\boxed{\Pi = \frac{1-\alpha}{\alpha}w\cdot\left[\frac{pA}{w}\right]^{\frac{1}{1-\alpha}}}$$

(b)

revenue = profit + labour cost.

$(pX) = (\Pi) + (wL)$

$$= \frac{1-\alpha}{\alpha}w\cdot\left[\frac{pA}{w}\right]^{\frac{1}{1-\alpha}} + w\left[\frac{pA}{w}\right]^{\frac{1}{1-\alpha}}.$$

$$= w\left[\frac{pA}{w}\right]^{\frac{1}{1-\alpha}}\left(\frac{1-\alpha}{\alpha} + 1\right)$$

$1-\alpha$ goes to profit $\quad \alpha$ goes to labour.

$\Pi \quad L.$

$(1-\alpha) : \alpha$

(c)

M capitalists

$\text{Labour}_{\text{supply}} = N \times 1 = N$

L_{demand} : profit max $= M \cdot \left[\frac{pA}{w}\right]^{\frac{1}{1-\alpha}}$

at eq. $L_S = L_D$

$$N = M \left[\frac{pA}{w}\right]^{\frac{1}{1-\alpha}}$$

$$\boxed{\left[\frac{pA}{w}\right]^{\frac{1}{1-\alpha}} = \frac{N}{M}}$$

(d)

$\uparrow A \Rightarrow \frac{p}{w}$ must fall since N/M is constant.

$w = 1$ hence p falls.

~~$\Rightarrow$ Consumer can buy more~~

also $\pi = \frac{1-\alpha}{\alpha} w \left[\frac{p}{w} A\right]^{\frac{1}{1-\alpha}}$

$\therefore \pi$ is unchanged since $A \uparrow$ and $p \downarrow$

But:

real wages $\uparrow$ + real π $\uparrow$

$\therefore$ gains to both in proportions $(1-\alpha) : \alpha$

(2)

Nash eq. : strategy profile such that each player's strategy is a best response to the strategies of the other players

Good prediction : yes if rational players + one pure Nash eq.

no if multiple ~~Nash~~ pure Nash eq. & mixed strategy Nash eq.

(a) $t=2$

Right : dom. strategy for column ~~(3 3 vs 2 and~~ (2 vs 3 and 0 vs 5)

∴ BR for row to play Up (3 vs 0)

∴ Pure strat. Nash equilibrium : {Up , Right}

(b) $t=4$

No pure strategy eq.

mixed : row plays Up w P_u
Need column indiff :

$$P_u(4) + (1-P_u)\cdot 0 = P_u \cdot 3 + (1-P_u)\cdot 5$$

$$P_u = 5 - 5P_u$$

$$6P_u = 5 \qquad \boxed{P_u = \frac{5}{6}}$$

Column plays Right P_R

$$P_R \cdot 3 + (1-P_R)\cdot 0 = 0\cdot P_R + (1-P_R)5$$

$$8P_R = 5$$

$$\boxed{P_R = \frac{5}{8}}$$

$\therefore$

mixed strategy Nash eq : $\{$ row plays Up with $p = \frac{5}{6}$, column plays Right with $p = \frac{5}{8}$ $\}$

(c) $t = 3$

Column weakly prefers Right. , to make row indifferent between Up & Down column should play Right $p = 5/8$

$\therefore$ ~~$\{$Up, Right$\}$~~

$\therefore$ Nash eq: $\{$ Up , Right w.p $= 5/8$ $\}$

~~surely just play~~

(?)

(3)

$u(w) = \ln(w)$

(a)

$$L_1 = \left[\tfrac{1}{2}, \tfrac{1}{2} ; 90, 40\right]$$

$$\mathbb{E}V(L_1) = \tfrac{1}{2}\cdot 90 + \tfrac{1}{2}\cdot 40 = \boxed{65}$$

$\boxed{\text{net return} = 1}$

$(65 - 64 = 1)$

$$RP = \mathbb{E}V - CE$$

$$u(CE) = \mathbb{E}u$$

$$\ln(CE) = \tfrac{1}{2}\ln 90 + \tfrac{1}{2}\ln 40$$

$$CE = e^{\frac{1}{2}\ln 90} \cdot e^{\frac{1}{2}\ln 40}$$

$$= 90^{\frac{1}{2}} \cdot 40^{\frac{1}{2}} = 60$$

$\boxed{RP = 5}$

Won't undertake since

$u(64) > \tfrac{1}{2}u(90) + \tfrac{1}{2}u(40)$

$\therefore$ optimal not to participate

(CE < initial wealth

$\therefore$ would pay not to have to play

$\therefore$ shouldn't play)

(b)

$$L_{S+J} = \left[\tfrac{1}{2}, \tfrac{1}{2} ; 77, 52\right]$$

$$\mathbb{E}V = \tfrac{1}{2}\cdot 77 + \tfrac{1}{2}\cdot 52 = \boxed{64.5}$$

$\boxed{\text{net return} = \tfrac{1}{2}}$

$$\ln(CE) = \tfrac{1}{2}\ln 77 + \tfrac{1}{2}\ln 52$$

$$CE = e^{\ln 77^{\frac{1}{2}} + \ln 52^{\frac{1}{2}}} = 77^{\frac{1}{2}} \cdot 52^{\frac{1}{2}} = \boxed{63.277}$$

$RP = 64.5 - 63.277 = \boxed{1.22}$

won't participate since CE < initial wealth.

$$L_{J+n} = \left[\tfrac{1}{2}, \tfrac{1}{2}; 64 + \tfrac{26}{n}, 64 - \tfrac{24}{n}\right]$$

participate if:

$$u(64) < \tfrac{1}{2}u\left(64 + \tfrac{26}{n}\right) + \tfrac{1}{2}u\left(64 - \tfrac{24}{n}\right)$$

$$\ln 64 < \ln\left(64 + \tfrac{26}{n}\right)^{\frac{1}{2}} + \ln\left(64 - \tfrac{24}{n}\right)^{\frac{1}{2}}$$

$$64 < \left(64 + \tfrac{26}{n}\right)^{\frac{1}{2}} \cdot \left(64 - \tfrac{24}{n}\right)^{\frac{1}{2}}$$

$$64^2 < \left(64 + \tfrac{26}{n}\right)\left(64 - \tfrac{24}{n}\right)$$

$$\cancel{64^2} < \cancel{64^2} + 64 \cdot \tfrac{26}{n} - 64\tfrac{24}{n} + -\tfrac{24 \cdot 26}{n^2}$$

$$0 < \frac{128}{n} - \frac{624}{n^2}$$

$$0 < n \cdot 128 - 624$$

$$n > \frac{39}{8} \qquad \boxed{n = 5}$$

When risk sharing with more people RP initially falls faster than expected net return

expected net return = $1, \frac{1}{2}, \frac{1}{3}, \dots, \frac{1}{n}, \dots$

risk premium = $5, 5 \cdot (\frac{1}{2})^2, \dots, 5(\frac{1}{n})^2, \dots$

(c)

$$\mathbb{E}U = \tfrac{1}{2}\ln\left(64 + \frac{26}{n}\right) + \tfrac{1}{2}\ln\left(64 - \frac{24}{n}\right)$$

max $\mathbb{E}U$

foc : $$\cancel{\tfrac{1}{2}}\frac{-26n^{-2}}{64 + 26n^{-1}} + \cancel{\tfrac{1}{2}}\frac{-24(-1)n^{-2}}{64 - 24n^{-1}} = 0$$

$$\frac{24/n^2}{64 - 24/n} = \frac{26/n^2}{64 + \frac{26}{n}}$$

$$\frac{24}{n^2}\left(64 + \frac{26}{n}\right) = \frac{26}{n^2}\left(64 - \frac{24}{n}\right)$$

$$24\cancel{n^2}\left(64 + \frac{26}{n}\right) = 26\left(64 - \frac{24}{n}\right)$$

$$24 \cdot 64 + \frac{624}{n} = 26 \cdot 64 - \frac{624}{n}$$

$$\cancel{\frac{26+24}{n} = 128}$$

$$\cancel{n = \frac{128}{50} = n}$$

$$\frac{624 + 624}{n} = 128$$

$$\cancel{n = \frac{64}{25} = 2.56}$$

$$n = \frac{39}{4} = 9.75$$

$$\therefore \boxed{n^* = 10}$$

- $n \doteq 10$ fall in net return dominates decrease in RP.

(4)

(a) Efficient outcome

- Symmetric information ∴ everyone knows the ~~value of both~~ quality of each bike
- High sells for £75 - £100
- Medium sells for £60 - £65
- Low does not sell

(whether med./high sell at top or bottom of range depends on if demand > supply or vice-versa)

- If demand > supply then high sells for £100 + med's for £65
- If supply > ~~supp~~ demand high sells for £75 + med's for £60

(b) All goods look same to buyer ∴ will sell at same price.

max WTP: $\frac{1}{3} \cdot 100 + \frac{1}{3} 65 + \frac{1}{3} \cdot 30 = £65$

at £65 sellers of high q bikes won't sell

∴ max WTP: $\frac{1}{2} \cdot 65 + \frac{1}{2} 30 = £47.5$

∴ sellers of medium q. won't sell

∴ max WTP = £30

∴ sellers of low q. won't sell

hence no bikes sell.

(c)

	Buyers value	Seller's value.
H	100	75
M	65 + 25	85
L	80	95

Buyers WTP: $\frac{1}{3}100 + \frac{1}{3}90 + \frac{1}{3}80 = 90$

~~Sellers~~ At £90 type L's won't sell (valued at £95)
$\therefore$ type L's don't sell.

Buyers WTP: $\frac{1}{2}100 + \frac{1}{2}90 = 95$
at £95 all bikes sell.

min. price for sellers = 85 (med.)

$$\boxed{85 \leq p < 95}$$

(d)

$75 \leq p < 85$ only type H sell
(inefficient).

(10)

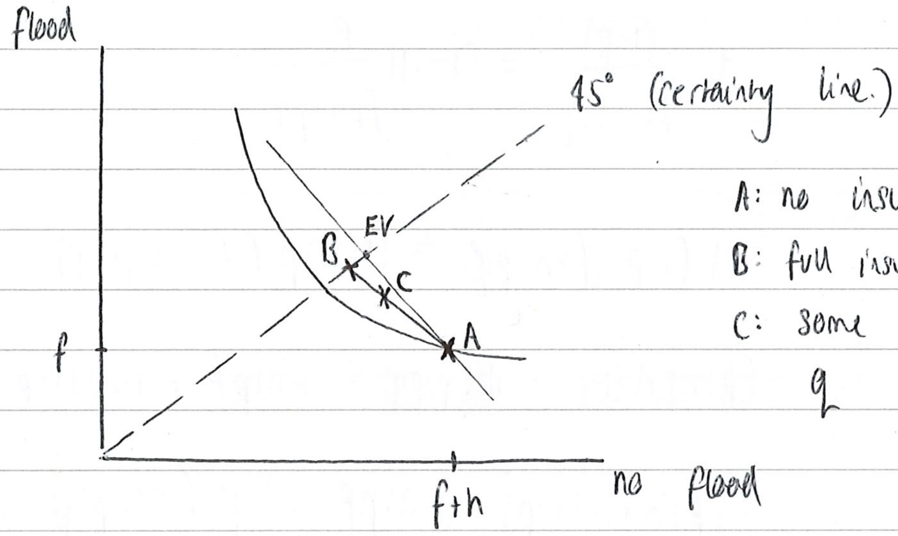
(a) $\mathbb{E}[\pi, (1-\pi); f+h-h, f+h]$

$= [\pi, (1-\pi); f, f+h]$

with insurance:

$= [\pi, (1-\pi); f+q-pq, f+h-pq]$

$= [\pi, (1-\pi); f+(1-p)q, f+h-pq]$



A: no insurance.

B: full insurance with $p > \pi$.

C: some level of insurence q

(b)

- Yes these are sufficient for risk aversion.

$$\mathbb{E}[u] = \pi \cdot u(f+(1-p)q) + (1-\pi) \cdot u(f+h-pq)$$

$$\frac{\partial E[u]}{\partial q} = \pi(1-p) \cdot u'(f+(1-p)q) + -p(1-\pi)\, u(f+h-pq)$$

at $q = h$

$= \pi - \cancel{\pi p} - p + \cancel{\pi p} = \pi - p$

$p > \pi \therefore$ -ve!!

$$\left.\frac{\partial \mathbb{E}[u]}{\partial q}\right|_{q=h} = \underbrace{u'(f+h-ph)}_{\substack{+ve \\ (u(\cdot) \text{ is increasing}) \\ \therefore u'(\cdot) > 0}} \underbrace{[\pi(1-p) - p(1-\pi)]}_{-ve \text{ (since } p > \pi)}$$

$\therefore \frac{\partial \mathbb{E}[u]}{\partial q} < 0$ $\therefore$ not optimised $\mathbb{E}[u]$ wrt to q, could improve $\mathbb{E}[u]$ by $\downarrow q$ $\therefore q < h$

(c)

$$\mathbb{E}[u] = \pi \ln(f + (1-p)q) + (1-\pi)\ln(f+h-pq)$$

foc: $$\frac{\partial \mathbb{E}[u]}{\partial q} = \pi \frac{(1-p)}{f+(1-p)q} + (1-\pi)\frac{-p}{f+h-pq}$$

$$\frac{\partial \mathbb{E}[u]}{\partial q} = 0 \quad @ \text{ ~~max~~. optimal.}$$

$$\pi \frac{(1-p)}{f+(1-p)q} = (1-\pi)\frac{p}{f+h-pq}$$

$$\pi(1-p)(f+h-pq^*) = (1-\pi)p(f+(1-p)q^*)$$

$$(f+h)\pi(1-p) - \pi(1-p)pq^* = (1-\pi)pf + (1-\pi)(1-p)p\,q^*$$

$$(f+h)\pi(1-p) - (1-\pi)pf = q^*\left(\pi(1-p)p + (1-\pi)(1-p)p\right)$$

$$q^* = \frac{(f+h)\pi(1-p) - (1-\pi)pf}{(1-p)p[\cancel{\pi} + 1 - \cancel{\pi}]}$$

$$q^* = \frac{(f+h)\pi(1-p) - (1-\pi)pf}{(1-p)p}$$

$$= \frac{f\pi(1-p) + h\pi(1-p) - (1-\pi)pf}{(1-p)p}$$

$$q^* = \frac{f(\pi(1-p) - (1-\pi)p) + \pi(1-p)h}{(1-p)p}$$

$$\boxed{q^* = \frac{f(\pi - p) + \pi(1-p)h}{(1-p)p}}$$

$q^* = 0$ when $f(\pi-p) + \pi(1-p)h = 0$
and $(1-p)p \neq 0$
(so fraction is well defined, $\therefore 0<p<1$)

$q^* = 0$ implies insurance line is tangential to ID curve at endowment $\therefore$ already maximising utility wrt to insurance.

(d)

$$q^* = \frac{f(\pi-p) + \pi(1-p)h}{(1-p)p}$$

$$\frac{\partial q^*}{\partial h} = \frac{\pi(1-p)}{(1-p)p} > 0 \quad \therefore \quad q^* \uparrow \text{ for } \uparrow h$$

$$\frac{\partial q^*}{\partial f} = \frac{\pi - p}{(1-p)p} < 0 \quad \therefore \quad q^* \downarrow \text{ for } \uparrow f$$

Why? $u(w) = \ln(w)$

- DARA: decreasing absolute risk aversion
  $A(w) = \frac{1}{w}$ $\therefore$ as wealth (f) increases risk aversion falls.

(CRRA also)

since unfavourable rate of ~~cover~~ p.

- All $1 > \frac{\partial q^*}{\partial h} > 0$ $\therefore$ insure more but at decreasing rate

Micro 2017

(1)

(a)

$$\max_{X,L} \quad u = X - \tfrac{1}{2}L^2 \quad \text{s.t.} \quad X = 2L^{1/2}$$

$$\max_{L} \; u = 2L^{1/2} - \tfrac{1}{2}L^2.$$

foc:

$$\frac{\partial u}{\partial L} = \cancel{\tfrac{1}{2}} \cdot \cancel{2} L^{-1/2} - \cancel{2} \cdot \cancel{\tfrac{1}{2}} L = 0$$

$$L = \frac{1}{L^{1/2}}$$

$$L^{3/2} = 1$$

$$\boxed{L = 1}$$

soc:

$$\frac{\partial^2 u}{\partial L^2} = -\tfrac{1}{2} L^{-3/2} < 0 \quad \forall L > 0$$

$\therefore$ maximum.

(b)

General comp. equilibrium = all markets are in equilibrium.

(c)

$$\max \; \pi = pX - wL \quad \text{s.t.} \quad X = 2L^{1/2}$$

$$\max_{L} \; p2L^{1/2} - wL$$

foc:

$$pL^{-1/2} - w = 0$$

$$\cancel{L = \frac{w}{p}} \qquad \boxed{L = \left(\frac{p}{w}\right)^2} \text{ labour demand.}$$

$$L^{-1/2} = \frac{w}{p}$$

$$\frac{1}{L^{1/2}} = \frac{w}{p} \qquad \frac{p}{w} = L^{1/2} \checkmark$$

$$\pi = pX - wL$$

$$\pi = p\,2\left[\left(\frac{p}{w}\right)^2\right]^{\frac{1}{2}} - w\left(\frac{p}{w}\right)^2$$

$$\pi = 2\,\frac{p^2}{w} - \cancel{w}\,\frac{p^2}{w^2}$$

$$\pi = \frac{p^2}{w}\,(2-1) \qquad \boxed{\pi = \frac{p^2}{w}}$$

households:

$$\max \quad U = X - \tfrac{1}{2}L^2 \qquad \text{s.t.} \quad pX = wL + \frac{p^2}{w}$$

$$MRS_{X,L} = -\frac{\frac{\partial u}{\partial L}}{\frac{\partial u}{\partial X}} = \frac{-L}{1} = L$$

@ max $\boxed{L = \frac{w}{p}}$ (labour supply)

at equilibrium:

$$\frac{w}{p} = \frac{p^2}{w^2} \qquad \boxed{1 = \frac{p^3}{w^3}} \Rightarrow \boxed{\frac{p}{w} = 1}$$

hence:

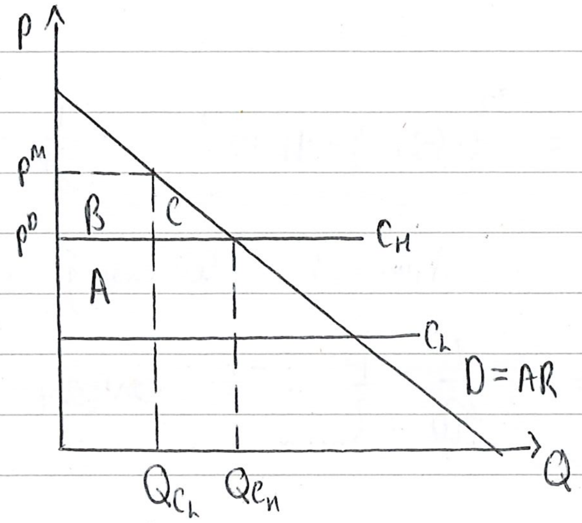
$$\boxed{\begin{array}{l} L_{supply} = 1 = L_{demand} \\ X = 2 \end{array}}$$

FTWE : Comp. equilibrium $\Rightarrow$ pareto efficient.

at $(X=2,\ L=1)$ no agent (consumer or firm) could have situation improved without making the other worse off.

- tax on income?
  - distortionary + would place a wedge between consumers MRS + producers MRT.

(2)



- If merger ↓ costs then
  $\Delta\Pi = A + B$
  $\Delta CS = -(B+C)$
  $\Delta PS = A + B$

  Δ social welfare $= A - C$.

  ∴ social welfare benefit providing $A > C$

  (area of rectangle vs triangles)

∴ if merger ↓ marginal cost by efficiency gains social welfare ↑!

- Notice here, however, consumer does not ~~win~~. gain since CS falls by $(B+C)$

- [Assumption : Bertrand - Nash ~~eq~~. comp. under duopoly implies pricing = mc]

(3)

$$u(w) = 2w^{\frac{1}{2}}$$

(a)

$$A(w) = -\frac{\cancel{2}\cdot\cancel{\frac{1}{2}}\cdot -\frac{1}{2}w^{-\frac{3}{2}}}{\cancel{2}\cdot\cancel{\frac{1}{2}}w^{-\frac{1}{2}}} = \frac{1}{2}w^{-\frac{3}{2}-\frac{1}{2}} = \frac{1}{2}w^{-1} = \frac{1}{2w}$$

$$\frac{\partial A(w)}{\partial w} = \cancel{\frac{1}{2}}\cdot(-2)\ \frac{1}{2}\cdot(-1)\ w^{-2} = -\frac{1}{2w^2} < 0$$

$\therefore$ $A(w)$ is decreasing in w.

$$R(w) = A(w)\cdot w = \frac{\cancel{1}}{2w}\ \frac{1}{2} = \text{constant}$$

(b)

$$L = \left[\tfrac{1}{2}, \tfrac{1}{2}; 104-40, \ 104+40\right]$$

$$u(CE) = EU[L]$$

$$2(CE)^{\frac{1}{2}} = \cancel{\tfrac{1}{2}}\cdot\cancel{2}(104-40)^{\frac{1}{2}} + \cancel{\tfrac{1}{2}}\cdot\cancel{2}(104+40)^{\frac{1}{2}}$$

$$CE^{\frac{1}{2}} = \tfrac{1}{2}(104-40)^{\frac{1}{2}} + \tfrac{1}{2}(104+40)^{\frac{1}{2}}$$

$$CE = 100$$

$$RP = EV - CE \qquad EV = 104$$

$$\boxed{RP = 100}$$

(c)

$RP(L) \approx \frac{1}{2}\sigma^2 A(w)$ ~~$A(w) = \frac{1}{2w}$~~

s.d ~~variance~~ of sam's is half that of Jones

$\sigma_S^2 = \frac{1}{2}\sigma_J^2$

$\therefore$ $RP(L_S)$ is ~~half~~ $\frac{1}{4}$ that of Jones

since $(s.d.)^2 =$ variance.

(4)

(a)

~~Signal may~~ Assuming perfectly comp. labour market then firms pay workers their productivity θ_L or θ_H

- If type H's can signal that they are type H's then they receive $w_S^* = \theta_H$, whereas if they can't firms pay some ~~expected~~ wage $w_{NS}^* = \mathbb{E}[\theta] = \lambda\theta_H + (1-\lambda)\theta_L$ where $\lambda =$ proportion of type H's

and $w_{NS}^* < w_S^*$ given $\lambda > 1$ and $\theta_L < \theta_H$.

- credible if signal is ~~inaccessible~~ sufficiently costly for type L's, hence not optimal for them to ~~compo~~ copy.

that is utility for type L's signalling and receiving $w = \theta_H$ is lower than utility of not signalling and receiving $w = \theta_L$

(b)

No signal:

$$w^* = \mathbb{E}[\theta] = \cancel{\frac{240}{240+180}} \ \frac{1}{3} \cdot 240 + \frac{2}{3} \cdot 180$$

$$\boxed{w^* = 200}$$

(c)

~~$U_H = 240 - 45 = 195$~~

~~$U_L = 180 - 55 =$~~

- Type H gain from signalling

$$\cancel{U_H} \ U_H = 240 - 45 = 195$$

- Type L no signal:

$$U_L = 180 - 0 = 180$$

- Type L copying signal:

$$U_L = 240 - 55 = 185$$

$$185 > 180$$

∴ not equilibrium since incentive is for type L to copy signal + earn higher wage

- Hence no separating equilibrium.

∴ pooling equilibrium: no signal + wage = 200

- Yes efficient, since signal = inefficient.

(10)

(a)

Observable $\Leftrightarrow$ Contractable

· hence only IR needs to bind

e_H:

$$\sqrt{w_{e_H}} - 3 = 0 \qquad \boxed{w_{e_H} = 9}$$

e_M:

$$\sqrt{w_{e_M}} - 2 = 0 \qquad \boxed{w_{e_M} = 4}$$

e_L:

$$\sqrt{w_{e_L}} - 1 = 0 \qquad \boxed{w_{e_L} = 1}$$

· which wage maximises profits?

$$\mathbb{E}[\Pi - w \mid e_H] = \frac{4}{5} \cdot 50 + \frac{1}{5} \cdot 10 - 9 = 33$$

$$\mathbb{E}[\Pi - w \mid e_M] = \frac{1}{5} \cdot 50 + \frac{4}{5} 10 - 4 = 14$$

$$\mathbb{E}[\Pi - w \mid e_L] = 0 \cdot 50 + 1 \cdot 10 - 1 = 9$$

$\therefore$ optimal to $\boxed{\text{induce } e_H}$!

(b)

Prefer e_M to e_H $\Rightarrow$

$$\frac{1}{5} V_2 + \frac{4}{5} V_1 - 2 \geq \frac{4}{5} V_2 + \frac{1}{5} V_1 - 3$$

$$1 \geq \left(\frac{4}{5} - \frac{1}{5}\right) V_2 + \left(\frac{1}{5} - \frac{4}{5}\right) V_1$$

$$1 \geq \frac{3}{5} V_2 - \frac{3}{5} V_1$$

$$\frac{5}{3} \geq (V_2 - V_1) \qquad \therefore \max(V_2 - V_1) = \frac{5}{3}$$

prefer e_M to e_L $\Rightarrow$

$$\tfrac{1}{5}V_2 + \tfrac{4}{5}V_1 - 2 \geq 0 \cdot V_2 + 1V_1 - 1$$

$$\tfrac{1}{5}V_2 + \left(\tfrac{4}{5} - \tfrac{5}{5}\right)V_1 \geq 1$$

$$\tfrac{1}{5}V_2 - \tfrac{1}{5}V_1 \geq 1$$

$$(V_2 - V_1) \geq 5$$

$$\therefore \min(V_2 - V_1) = 5$$

To choose e_M it must be the case that

① $(V_2 - V_1) < \frac{5}{3}$ (other wise e_H would be optimal)

and

② $(V_2 - V_1) > 5$ (other wise e_L would be optimal)

if condition ① holds then ② does not, hence e_L is optimal

if condition ② holds then ① does not, hence e_H is optimal.

$\therefore$ no such contract where e_M is optimal.

(c)

(i)

Implement e_L simply by ~~to~~ satisfying IR constraint:

$$\sqrt{w_{e_L}} - 1 \geq 0$$

Since then incentive for worker to worker, but no incentive to exert effort.

Optimal for owner that the constraint binds (max Π)

$\therefore \quad \boxed{w^*_{e_L} = 1} \qquad \boxed{\mathbb{E}[\pi | e_L] = 10 - 1 = 9}$

(ii)

Satisfy IR + IC constraints:

IR: $\sqrt{\frac{4}{5}V_2} + \frac{1}{5}V_1 - 3 \geq 0$

① $4V_2 + V_1 \geq 15$

IC:

$$\frac{4}{5}V_2 + \frac{1}{5}V_1 - 3 \geq 0 \cdot V_2 + 1 \cdot V_1 - 1$$

$$\frac{4}{5}V_2 + \frac{1}{5}V_1 - \frac{5}{5}V_1 \geq 2$$

② $4V_2 - 4V_1 \geq 10$

4x① + 4x②:

$$\cancel{4V_2}\ 16V_2 + 4V_1 + 16V_2 - 4V_1 \geq 4 \times 15 + 10 = 70$$

$$32V_2 \geq 70$$

$V_2 \geq \frac{35}{16}$ $\quad w_2 = \frac{625}{256} = 2.44$ $\quad \boxed{w_2 = 4.785}$

$V_1 \geq 15 - 4 \cdot \frac{25}{16} = \cancel{8.75}\ 6.25$

$4 \times ① + ②$:

$$16v_2 + 4v_1 + 4v_2 - 4v_1 \geq \underbrace{15 \times 4 + 10}_{= 70}$$

$$20v_2 \geq 70$$

$$v_2 \geq \frac{7}{2} \qquad v_1 \geq 15 - 4 \cdot \frac{7}{2} = 1$$

bind at optimal for firm, hence,

$$\boxed{w_2 = \frac{49}{4} = 12.25 \qquad w_1 = 1}$$

firm needs to max. $\mathbb{E}[\Pi]$

- when inducing e_H :

$$\mathbb{E}[\Pi - w \mid e_H] = 50 \cdot \frac{4}{5} + 10 \cdot \frac{1}{5} - 12.25 \frac{4}{5} - 1 \cdot \frac{1}{5}$$

$$= 32$$

$$\mathbb{E}[\Pi - w \mid e_H] = 32 > 9 = \mathbb{E}[\Pi - w \mid e_L]$$

hence induce e_H with contract:

$w = 1$ if small harvest and

$w = 12.25$ if large harvest

(d) Agency cost:

$$\mathbb{E}[w \mid e_H] = \frac{4}{5} \cdot 12.25 + \frac{1}{5} \cdot 1 = 10$$

wage w. certainty = CE

$\mathbb{E}[w \mid e_H] = EV$

RP = EV − CE

$$10 - 9 = 1$$

Agency cost = 1

- a variable wage which ensures agent reservation utility > fixed wage which does the same since Agent = risk averse

∴ Agency cost = risk premium.